STATISTICAL DEPENDENCE IN MARKOV RANDOM FIELD MODELS

by

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ABSTRACT

Statistical models based on Markov random fields present a flexible means for modeling statistical dependencies in a variety of situations including, but not limited to, spatial problems with observations on a lattice. The simplest of such models, sometimes called "auto-models" are formulated from sets of conditional one-parameter exponential family densities or mass functions. Despite the attractive nature of these models for dealing with complex dependence structures, their application has been hindered by a lack of interpretability relative to the manner in which dependencies are represented. In particular, while the parameters that embody dependence are nicely isolated in these models, the meaning of numerical values of those parameters as representing dependence of varying strengths has been poorly understood. In addition, it is known that dependence parameters that are "too large" lead to un-interpretable or even degenerate behavior in data sets simulated from models having such parameters. The objectives of this article are to identify a concept of dependence that is generally applicable to Markov random field models based on one-parameter exponential families, and to demonstrate the relation between a quantification of this concept of dependence and the dependence parameters in models. It is then possible to both quantify the strength of statistical dependencies represented by particular numerical values of dependence parameters, and delineate ranges of those parameters that lead to separable interpretations of large-scale model components as marginal mean structure and small-scale components as additional statistical dependence.